LABEL-EFFICIENT BAYESIAN **ASSESSMENT OF BLACK-BOX CLASSIFIERS**

- **Department of Computer Science**
 - University of California, Irvine

Committee members:

Chancellor's Professor Padhraic Smyth, chair **Assistant Professor Stephan Mandt Professor Mark Steyvers**

Label-efficient Bayesian Assessment of Black-box Classifiers

Disi Ji

November 18, 2020





Label-efficient Bayesian Assessment of Black-box Classifiers







Label-efficient Bayesian Assessment of Black-box Classifiers





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by Sian Townson November 06, 2020

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As banks increasingly deploy artificial intelligence tools to make credit decisions, they are having to revisit an unwelcome fact about the practice of lending: Historically, it has been riddled with biases against protected characteristics, such as race, gender, and sexual orientation. Such biases are evident in institutions' choices in terms of who gets credit and on what terms. In this context, relying on algorithms to make credit decisions instead of deferring to human judgment seems like an obvious fix. What machines lack in warmth, they surely make up for in objectivity, right?

The lessons we all must learn from the A-levels algorithm debacle

education admissions

Unless action is taken, similar systems will suffer from the same mistakes. And the consequences could be dire





Label-efficient Bayesian Assessment of Black-box Classifiers

finance decisions







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SECTIONS ECONOMICS COLLECTIONS ARTIFICIAL INTELLIGENCE STATISTICS







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The United States of Risk Assessment: The Machines Influencing Criminal Justice Decisions

In every state, assessment tools help courts decide certain cases or correctional officers determine the supervision and programming an offender receives. But the tools each state uses varies widely, and how they're put into practice varies even more.

By Rhys Dipshan, Victoria Hudgins and Frank Ready | July 13, 2020 at 07:00 AM







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- Assessment machine learning models independently from the training procedures
 - legal requirement, build consumers' trust in model predictions
 - distribution change at deployment time:
 - Iabel shift [Lipton et al. 2018]
 - corruptions and perturbations [Hendrycks et al. 2019, Ovadia et al. 2019b]
 - models' inability to generalize [Recht et al. 2019]

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Do ImageNet Classifiers Generalize to ImageNet?

Benjamin Recht, Rebecca Roelofs, Ludwig Schmidt, Vaishaal Shankar

We build new test sets for the CIFAR-10 and ImageNet datasets. Both benchmarks have been the focus of intense research for almost a decade, raising the danger of overfitting to excessively reused test sets. By closely following the original dataset creation processes, we test to what extent current classification models generalize to new data. We evaluate a broad range of models and find accuracy drops of 3% – 15% on CIFAR-10 and 11% – 14% on ImageNet. However, accuracy gains on the original test sets translate to larger gains on the new test sets. Our results suggest that the accuracy drops are not caused by adaptivity, but by the models' inability to generalize to slightly "harder" images than those found in the original test sets.











































. . .

- And how much **confidence** should we have in this assessment?
- How to **increase our confidence** given the labeling budget?







Confidence of M

























ROAD MAP

Bayesian assessment

1. Quantify uncertainty of assessment with Bayesian methods, with a set of labeled data

active Bayesian assessment

2. Reduce uncertainty of assessment, with actively labeled data selected from a pool of unlabeled data

Label-efficient Bayesian Assessment of Black-box Classifiers

assess with **unlabeled** data

3. **Reduce uncertainty** of assessment, by leveraging both labeled and unlabeled data

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Input: X

Input: X

Input: X

label: y Dog

Label-efficient Bayesian Assessment of Black-box Classifiers

Dog

Label-efficient Bayesian Assessment of Black-box Classifiers

Dog

Accuracy
$$\theta = \mathbb{E}_{p(x,y)} \mathbb{1}(y = \hat{y})$$

Empirical accuracy $\hat{\theta} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}(y_i = \hat{y}_i)$

Deep neural networks are miscalibrated [<i>Guo et al. 2017</i>]		1.0
		0.9
e.g. ResNet-110 on CIFAR-100		0.8
Reliability diagram		0.7
Expected calibration error (ECE)	acy	0.6
	ccura	0.5
	A	0.4^{-1}
		0.3
		0.2
		0.1
		0.0

Reliability diagram for ResNet-110 on CIFAR-100

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Reliability diagram for ResNet-110 on CIFAR-100 computed with 10,000 data points

$$= \mathbb{E}_{p(x,y)} \mathbb{1}(y = \hat{y})$$
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Accuracy of the k-th predicted class:

$$\theta_k = \mathsf{Beta}(\alpha_k, \beta_k), k = 1, 2, \cdots, K$$

Accuracy

classwise accuracy for ResNet-110 on CIFAR-100

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Accuracy

classwise accuracy for ResNet-110 on CIFAR-100

Label-efficient Bayesian Assessment of Black-box Classifiers

Accuracy of the *b*-th bin: $\theta_b = \text{Beta}(\alpha_b, \beta_b), b = 1, 2, \cdots, B$

binwise accuracy for ResNet-110 on CIFAR-100

BAYESIAN ASSESSMENT: HOW CALIBRATED

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BAYESIAN ASSESSMENT: HOW CALIBRATED

BAYESIAN ASSESSMENT: HOW CALIBRATED

classwise accuracy vs classwise ECE ResNet-110 on CIFAR-100

Label-efficient Bayesian Assessment of Black-box Classifiers

motorcycle pickup_truck

Accuracy of the *b*-th bin of the *k*-th predicted class:

 $\theta_{kb} = \text{Beta}(\alpha_{kb}, \beta_{kb})$

 $k = 1, 2, \dots, K; \quad b = 1, 2, \dots, B$

SUMMARY

 ✓ How accurate? ✓ How calibrated? ● How fair? ●other metrics
And how much confidence should we have in this assessment?
How to increase our confidence given the labeling budget?

Label-efficient Bayesian Assessment of Black-box Classifiers

classwise accuracy

Accuracy of the k-th predicted class:

 $\theta_k = \text{Beta}(\alpha_k, \beta_k), k = 1, 2, \cdots, K$

binwise accuracy(ECE)

Accuracy of the *b*-th bin:

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Classwise ECE

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RVINE

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	Partition of Input space
classwise accuracy	
Accuracy of the k-th predicted class: $\theta_k = \text{Beta}(\alpha_k, \beta_k), k = 1, 2, \dots, K$	Predicted class
binwise accuracy(ECE) Accuracy of the <i>b</i> -th bin: $\theta_b = \text{Beta}(\alpha_b, \beta_b), b = 1, 2, \dots, B$	Model score
Classwise ECE	
Accuracy of the <i>b</i> -th bin of the <i>k</i> -th predicted class: $\theta_{kb} = \text{Beta}(\alpha_{kb}, \beta_{kb})$ $k = 1, 2, \dots, K; b = 1, 2, \dots, B$	Predicted class X model score

Label-efficient Bayesian Assessment of Black-box Classifiers







IRVINE

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Label-efficient Bayesian Assessment of Black-box Classifiers



g = kb





Performance metrics to estimate $\theta = (\theta_0, \theta_1, \dots, \theta_G)$ **Estimation**: estimate model performance across all groups[1] e.g. minimize RMSE = $(\sum p_g (\hat{\theta}_g - \theta_g^*)^2)^{\frac{1}{2}}$ Identification: identify extreme groups, e.g. least accurate, least calibrated • e.g. identify $\hat{g} = \arg \max_{g} \theta_{g}$ **Comparison**: compare performance between two groups • e.g. $\theta_0 > \theta_1$?

[1] Sawade et al. [2010] and Kumar and Raj [2018] use importance sampling and stratified sampling respectively to allocate labeling resources among different groups.









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ers







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MULTI-ARMED BANDIT PROBLEMS

- reward probabilities of each arm are not told in advance
- objective: maximize cumulative reward
- exploration-exploitation trade-off
- **budget**: decide when to switch from more exploration to more exploitation
- Sequential decision making









ONLINE DECISION SYSTEM



Label-efficient Bayesian Assessment of Black-box Classifiers





IRVINE

ONLINE DECISION SYSTEM

the gambler's decision process



Label-efficient Bayesian Assessment of Black-box Classifiers





IRVINE

ONLINE DECISION SYSTEM







$\mathbf{N}=\mathbf{0}$



Label-efficient Bayesian Assessment of Black-box Classifiers





RVINE

$\mathbf{N} = \mathbf{0}$







RVINE

active Bayesian assessment





















	Assessment Task	p(heta)	$q_{ heta}(z g)$	r(z g)
Estimation	Groupwise Accuracy	$\theta_g \sim \text{Beta}(\alpha_g, \beta_g)$	$z \sim \operatorname{Bern}(\theta_g)$	$p_g \cdot (\operatorname{Var}(\hat{ heta}_g \mathcal{L}) - \operatorname{Var}(\hat{ heta}_g \{\mathcal{L}, z\}))$
	Confusion $Matrix(g = k)$	$\theta_{\cdot k} \sim \mathrm{Dirichlet}(\alpha_{\cdot k})$	$z \sim \operatorname{Multi}(\theta_k)$	$p_k \cdot (\operatorname{Var}(\hat{ heta}_k \mathcal{L}) - \operatorname{Var}(\hat{ heta}_k \{\mathcal{L}, z\}))$
Identification	Least Accurate Group	$\theta_g \sim \text{Beta}(\alpha_g, \beta_g)$	$z \sim \operatorname{Bern}(\theta_g)$	$-\widetilde{ heta}_g$
	Least Calibrated Group	$\theta_{gb} \sim \text{Beta}(\alpha_{gb}, \beta_{gb})$	$z \sim \text{Bern}(\theta_{gb})$	$\sum_{b=1}^{B} p_{gb} \left \widetilde{ heta}_{gb} - s_{gb} \right $
	Most Costly $Class(g = k)$	$\theta_{\cdot k} \sim \mathrm{Dirichlet}(\alpha_{\cdot k})$	$z \sim \operatorname{Multi}(\theta_k)$	$\sum_{j=1}^{K} c_{jk} \widetilde{ heta}_{jk}$
Comparison	Accuracy Comparison	$\theta_g \sim \text{Beta}(\alpha_g, \beta_g)$	$z \sim \operatorname{Bern}(\theta_g)$	$\lambda \{\mathcal{L},(g,z)\}$





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EXPERIMENTS: MATERIAL

- Difference mode, varying size and number of classes
- Kudos to Robby for training the classification models

	Mode	Size	Classes	Model
CIFAR-100	Image	10K	100	ResNet-110
ImageNet	Image	50K	1000	$\operatorname{ResNet-152}$
SVHN	Image	26K	10	$\operatorname{ResNet-152}$
20 Newsgroups	Text	$7.5 \mathrm{K}$	20	$BERT_{BASE}$
DBpedia	Text	70K	14	$\operatorname{BERT}_{\operatorname{BASE}}$







EXAMPLE: IDENTIFY THE LEAST ACCURATE CLASS

Percentage of labeled samples needed to identify the least accurate classes

Dataset	Top m	UPrior (baseline)	IPrior (our work)	IPrior+TS (our work)	
CIFAR-100	1	81.1	83.4	24.9	
	10	99.8	99.8	55.1	
ImageNet	1	96.9	94.7	9.3	Dropped by 90%
	10	99.6	98.5	17.1	
SVHN	1	90.5	89.8	82.8	
	3	100.0	100.0	96.0	
20 Newsgroups	1	53.9	55.4	16.9	
	3	92.0	92.5	42.5	
DBpedia	1	8.0	7.6	11.6	
	3	91.9	90.2	57.1	





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	3	92.0	92.5	42.5	
DBpedia	1	8.0	7.6	11.6	
	3	91.9	90.2	57.1	

We obtained similar performance gain across multiple datasets, prediction models, and assessment tasks





DISCUSSION

Other Bayesian active learning method to TS?

- Comparisons with alternative active learning algorithms
- e.g. Epsilon-greedy, Bayesian upper-confidence bound
- Thompson sampling is broadly more reliable and more consistent
- TS is not designed for exploration-only problems (best arm identification) Comparisons between TS and top-two TS
- - TS and TTTS gave very similar performance
- **Sensitivity analysis** for hyperparameters
 - appears to be relatively robust to the prior strength





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active Bayesian assessment

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assess with **unlabeled** data

3. Reduce uncertainty of assessment, by leveraging both **labeled and unlabeled data**









IS THE CLASSIFIER REALLY UNFAIR? Classified as negative

model score of a binary classifier M

Equality of opportunity: equal TPR across different groups^[1]

Due to small sample size, the estimated TPR is noisy!

[1] "Equality of Opportunity in Supervised Learning". Hardt, Price & Srebro. NeurIPS 2016.





- people who pay back their loan, have an equal opportunity of getting the loan in the first place"









MODEL FAIRNESS METRICS WITH UNCERTAINTY **Classified as negative Classified as positive** score of a classifier M



Δ TPR between female and male









MODEL FAIRNESS METRICS WITH UNCERTAINTY

Classified as negative

score of a classifier M



$\Delta {\rm TPR}$ between female and male

Label-efficient Bayesian Assessment of Black-box Classifiers

Classified as positive









MODEL FAIRNESS METRICS WITH UNCERTAINTY

Classified as negative

score of a classifier M



$\Delta {\rm TPR}$ between female and male

Label-efficient Bayesian Assessment of Black-box Classifiers

Classified as positive



Q: The uncertainty is high! How to reduce it? **A:** Collect more data! Labeled or **unlabeled!**





HIGH UNCERTAINTY FOR REAL-WORLD DATA



whether income exceeds \$50,000 per year

whether the individual has subscribed to a term deposit account or not

frequency-based estimates of the difference in true positive rate (TPR)

Label-efficient Bayesian Assessment of Black-box Classifiers

COMPAS (Correctional Offender Management Profiling for Alternative Sanctions) risk assessment tool for recidivism







HOW MANY LABELED DATA DO I NEED TO COLLECT?

- Simulation:
 - ▶ p(g=0) = 20%
 - groupwise positive rates p(y = 1) are both 20%
 - the true groupwise TPRs are 95% and 90%.
- Compute frequentist estimation of Δ TPR for 10000 times

Label-efficient Bayesian Assessment of Black-box Classifiers





RVINE

HOW MANY LABELED DATA DO I NEED TO COLLECT?

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Label-efficient Bayesian Assessment of Black-box Classifiers





RVINE

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- the true groupwise TPRs are 95% and 90%.
- Compute frequentist estimation of Δ TPR for 10000 times



Dataset	Test Size	G	p(g=0)	p(y=1)
Adult	10054	gender, race	0.68, 0.86	0.25
Bank	13730	age	0.45	0.11
German	334	age, gender	0.79,0.37	0.17
Compas-R	2056	gender, race	0.7, 0.85	0.69
$\operatorname{Compas-VR}$	1337	gender, race	0.8, 0.34	0.47
Ricci	40	race	0.65	0.50







Label-efficient Bayesian Assessment of Black-box Classifiers

RVINE




Label-efficient Bayesian Assessment of Black-box Classifiers

REDUCE UNCERTAINTY OF FAIRNESS WITH MORE UNLABELED DATA

Classified as positive

performance on unlabeled data



































- #labeled data in some groups is small: use Hierarchical Bayesian calibration to share statistical strength among groups
- Variance of the estimates is high: augment with unlabeled data by predicting labeling outcomes with BC
- Calibration model: any parametric calibration model, e.g. Beta calibration











EXAMPLE: ASSESS DELTA TPR OF COMPAS RECIDIVISM



With **10** labeled data and ~**2000** unlabeled data, error in estimating TPR is **5%** for our method versus 20% with only labeled data





EXAMPLE: ASSESS DELTA TPR OF COMPAS RECIDIVISM



With **10** labeled data and ~**2000** unlabeled data, error in estimating TPR is **5%** for our method versus 20% with only labeled data

We obtained similar performance gain across multiple dataset-attribute combinations, prediction models, and fairness metrics





DISCUSSION

- bias-variance tradeoff
- potential **error in the calibration mapping** (e.g., due to misspecification of the parametric form of the calibration function) to **error in the estimate of \Delta itself**

Lemma 4.5.1. Given a prediction model M and score distribution P(s), let $f_q(s; \phi_q)$: $[0,1] \rightarrow [0,1]$ denote the calibration model for group g; let $f_g^*(s) : [0,1] \rightarrow [0,1]$ be the optimal calibration function which maps $s = P_M(\hat{y} = 1|g)$ to P(y = 1|g); and Δ^* is the true value of the metric. Then the absolute error of the expected estimate w.r.t. ϕ can be bounded as: $|\mathbb{E}_{\phi}\Delta - \Delta^*| \leq \|\bar{f}_0 - f_0^*\|_1 + \|\bar{f}_1 - f_1^*\|_1$, where $\bar{f}_g(s) = \mathbb{E}_{\phi_g}f_g(s;\phi_g), \forall s \in [0,1]$, and $\|\cdot\|_1$ is the expected L1 distance w.r.t. P(s|g).





DISCUSSION

Calibration of the posterior probability

- > a perfectly calibrated 95% credible interval would have 95% coverage.
- generally not far from 95% there is room for improvement (model misspecification)

How about other calibration models?

- comparisons with an alternative calibration model, i.e. LLO calibration
- two calibration methods tends to be very similar

Is the hierarchical structure necessary?

- ablation study by comparing with non-hierarchical Bayesian calibration
- Hierarchical structure helps with avoiding occasional catastrophic errors

Sensitivity analysis for the calibration priors

robust to the settings of prior variances

Label-efficient Bayesian Assessment of Black-box Classifiers

val would have 95% coverage. for improvement (model misspecification)

on model, i.e. LLO calibration similar

erarchical Bayesian calibration coccasional catastrophic errors

UCIE





Bayesian assessment

1. Quantify uncertainty of assessment with Bayesian models, with a set of **labeled data**





Label-efficient Bayesian Assessment of Black-box Classifiers

assess with **unlabeled** data

3. Reduce uncertainty of assessment, by leveraging both labeled and unlabeled data









Bayesian estimation of performance metrics

- (1) accuracy, reliability diagram, ECE
- (2) Performance difference
- (3) Confusion matrix, misclassification cost

Use self-assessment as informative priors

Bayesian assessment

1. Quantify uncertainty of assessment with Bayesian models, with a set of **labeled data**





unlabeled data



[Ji, Logan, Smyth, Steyvers 2019 ICML UDL]

Label-efficient Bayesian Assessment of Black-box Classifiers

active Bayesian assessment

2. Reduce uncertainty of assessment, with actively labeled data selected from a pool of

assess with **unlabeled** data

3. Reduce uncertainty of assessment, by leveraging both labeled and unlabeled data









Bayesian estimation of performance metrics

(1) accuracy, reliability diagram, ECE



[Ji, Logan, Smyth, Steyvers 2019 ICML UDL] [Ji, Logan, Smyth, Steyvers 2021 AAAI?]

Label-efficient Bayesian Assessment of Black-box Classifiers

Developed active assessment framework for (1) estimation of model performance;

assess with **unlabeled** data

3. Reduce uncertainty of assessment, by leveraging both labeled and unlabeled data









Bayesian estimation of performance metrics

(1) accuracy, reliability diagram, ECE



[Ji, Logan, Smyth, Steyvers 2019 ICML UDL]

Label-efficient Bayesian Assessment of Black-box Classifiers

- Developed active assessment framework for (1) estimation of model performance;

[Ji, Logan, Smyth, Steyvers 2021 AAAI?]

- (1) Proposed a comprehensive Bayesian treatment of fairness assessment
- (2) Developed a new hierarchical Bayesian model to leverage information from both unlabeled and labeled examples

assess with **unlabeled** data

3. Reduce uncertainty of assessment, by leveraging both labeled and unlabeled data



[Ji, Smyth, Steyvers 2020 NeurIPS]









LIST OF PUBLICATIONS

- Bayesian Evaluation of Black-Box Classifiers. [Ji, Logan, Smyth, Steyvers ICML UDL 2019]
- Steyvers NeurIPS 2020]
- Active Bayesian Assessment for Black-Box Classifiers. [Ji, Logan, Smyth, Steyvers AAAI 2021?]

Label-efficient Bayesian Assessment of Black-box Classifiers

Can I Trust My Fairness Metric? Assessing Fairness with Unlabeled Data and Bayesian Inference. [Ji, Smyth,







LIST OF PUBLICATIONS

- Bayesian Evaluation of Black-Box Classifiers. [Ji, Logan, Smyth, Steyvers ICML UDL 2019]
- **Steyvers NeurIPS 2020**]
- Active Bayesian Assessment for Black-Box Classifiers. [Ji, Logan, Smyth, Steyvers AAAI 2021?]

Automated diagnosis of Leukemia with cytometry data analysis

- Mondrian Processes for Flow Cytometry Analysis. [Ji, Nalisnick, Smyth NeurIPS ML4H 2017]
- Bayesian Trees for Automated Cytometry Data Analysis. [Ji, Nalisnick, Qian, Scheuermann, Smyth MLHC 2018]
- Learning Discriminative Gating Representations for Cytometry Data. [Ji, Putzel, Qian, Scheuermann, Bui, Wang, Smyth ICML Workshop on Computational Biology 2019]
- Optimization of Automated Gating for Clinical Diagnosis using Discriminative Gates. [Ji, Putzel, Qian, Scheuermann, Bui, Wang, Smyth Cytometry: Part A 2019]

Label-efficient Bayesian Assessment of Black-box Classifiers

Can I Trust My Fairness Metric? Assessing Fairness with Unlabeled Data and Bayesian Inference. [Ji, Smyth,



























































EXPERIMENTS: BAYESIAN ESTIMATION OF ECE







USE SELF-ASSESSMENT AS INFORMATIVE PRIOR







BAYESIAN RELIABILITY DIAGRAMS









COMPARISONS WITH ALTERNATIVE ACTIVE LEARNING ALGORITHMS³



Label-efficient Bayesian Assessment of Black-box Classifiers

Least accuracy classes





COMPARISONS BETWEEN TS AND TTTS



Label-efficient Bayesian Assessment of Black-box Classifiers

Least accuracy classes











COMPARISONS BETWEEN IPRIOR+TS AND UPRIOR+TS



Label-efficient Bayesian Assessment of Black-box Classifiers

Least accuracy classes



SENSITIVITY ANALYSIS FOR HYPERPARAMETERS



Least accuracy classes





$\Delta FPR ESTIMATION$







(FAIRNESS) CALIBRATION OF THE POSTERIOR PROBABILITY®

Table 4.4: Calibration Coverage of Posterior Credible Intervals Comparison, across 1000 runs of labeled samples of different sizes n_L for 10 different dataset-group combinations (rows). Estimation methods are BC (Bayesian-Calibration) and BB (beta-bernoulli). Trained model is multi-layer perceptron.

	$n_L = 10$			$n_{L} = 20$			$n_L = 40$			$n_L =$	100
Group	BC	BB	_	BC	BB		BC	BB		BC	BB
Adult, Race	99.9	97.7		98.6	93.5		96.2	93.2		92.3	95.3
Adult, Gender	100.0	96.4		99.7	95.5		99.2	94.9		96.8	95.5
Bank, Age	99.4	98.7		98.8	98.5		98.0	96.4		93.7	95.3
German, age	99.9	98.8		99.6	98.1		99.0	98.3		96.9	98.3
German, Gender	99.1	97.4		99.1	97.4		97.7	96.4		94.6	97.8
Compas-R, Race	99.3	98.8		99.4	97.2		99.1	96.7		99.3	96.6
Compas-R, Gender	99.3	97.7		99.3	97.0		98.6	95.9		97.6	96.5
Compas-VR, Race	99.6	100.0		98.6	97.8		97.9	95.2		97.5	93.1
Compas-VR, Gender	96.3	97.2		94.3	96.5		95.4	96.1		95.8	97.1
Ricci, Race	93.2	99.7		91.4	99.7						



COMPARISONS WITH LLO CALIBRATION

		Multi-la	yer Perceptron	Logist	ic Regression	Rande	om Forest	Gaussian Naive Bayes		
Group	n	BC	LLO	BC	LLO	BC	LLO	BC	LLO	
Adult	10	3.9	3.8	2.9	2.8	3.2	3.2	3.6	3.5	
Race	100	3.5	3.4	3.2	3.1	3.1	2.9	2.8	2.4	
	1000	1.6	2.3	1.7	2.0	1.4	1.5	1.4	1.6	
Adult	10	5.1	5.1	2.2	2.3	4.8	4.7	5.4	5.0	
Gender	100	4.4	4.3	1.9	2.0	4.1	3.7	2.7	2.7	
	1000	1.6	2.2	1.1	1.0	2.0	1.5	1.1	1.1	
Bank	10	2.5	2.3	1.4	1.2	1.0	0.9	1.7	1.7	
Age	100	2.0	2.0	1.2	1.2	0.9	0.9	1.1	1.2	
	1000	1.1	1.2	0.7	0.7	0.5	0.5	0.8	0.9	
German	10	5.0	4.6	8.7	8.0	8.2	7.5	11.5	10.7	
age	100	3.9	4.1	3.8	4.7	4.3	4.0	4.2	6.0	
	200	3.1	3.9	3.3	4.2	3.3	3.1	3.5	6.0	
German	10	8.2	6.4	6.3	5.0	8.6	6.9	6.5	5.3	
Gender	100	5.4	5.1	3.7	3.6	4.8	4.5	2.8	3.1	
	200	3.0	3.4	2.9	2.8	2.9	3.1	2.2	2.9	
Compas-R	10	4.2	4.6	4.8	5.2	2.4	2.5	8.4	8.2	
Race	100	2.8	4.4	3.4	4.8	1.8	1.4	6.0	5.6	
	1000	1.6	5.0	1.6	4.4	1.2	1.1	1.8	2.9	
Compas-R	10	5.0	4.3	3.8	3.9	4.4	4.1	13.7	13.0	
Gender	100	3.3	2.7	2.6	2.3	2.7	2.8	8.0	7.4	
	1000	1.4	2.1	1.3	1.3	1.4	3.0	1.8	2.4	
Compas-VR	10	4.0	3.9	4.4	4.7	2.4	2.9	6.5	6.4	
Race	100	3.1	2.8	3.4	3.3	2.0	2.1	3.7	3.6	
	1000	0.8	1.5	0.8	0.8	0.8	2.5	0.9	1.8	
Compas-VR	10	5.4	4.8	5.3	5.2	6.3	8.2	9.8	9.0	
Gender	100	3.4	3.0	3.1	3.3	4.4	5.4	4.5	4.2	
	1000	0.9	1.2	0.9	1.5	1.0	1.7	0.9	0.9	
Ricci	10	14.6	14.2	7.9	8.1	2.1	2.0	1.6	2.1	
Race	20	9.8	13.6	7.1	6.6	1.5	1.6	2.1	2.5	
	30	6.5	12.1	4.6	4.2	1.1	1.4	2.0	2.3	

Label-efficient Bayesian Assessment of Black-box Classifiers





IRVINE

IS THE HERARCHICAL STRUCTURE NECESSARY?

		Mult	i-layer Per	rceptron	Logis	stic Regre	ssion	Ra	ndom For	rest	Gaus	ssian Naive Bayes	
Group	n	BB	NHBC	BC	BB	NHBC	BC	BB	NHBC	BC	BB	NHBC	BC
Adult	10	18.4	3.2	3.9	18.8	2.7	2.9	18.1	2.8	3.2	18.9	4.5	3.6
Race	20	16.1	3.3	4.4	16.7	2.9	3.4	16.3	3.0	3.7	16.8	4.1	3.7
	40	13.1	2.8	4.5	14.0	2.9	3.7	14.4	2.9	3.8	14.4	3.7	3.3
	100	8.6	2.7	3.5	9.2	3.0	3.2	9.0	2.6	3.1	9.6	2.4	2.8
	1000	2.5	1.4	1.6	2.3	2.1	1.7	2.1	0.7	1.4	2.3	1.8	1.4
Adult	10	17.4	4.1	5.1	16.3	2.6	2.2	17.3	5.3	4.8	16.3	7.2	5.4
Gender	20	12.9	4.4	5.1	12.2	2.6	2.2	12.4	5.3	4.9	11.6	6.7	4.5
	40	9.0	4.1	4.9	9.2	2.5	2.1	9.6	5.1	4.5	9.7	6.3	3.9
	100	5.4	3.1	4.4	5.5	2.0	2.0	5.9	4.7	4.1	6.0	4.8	2.7
	1000	1.9	1.4	1.6	1.7	1.0	1.1	1.5	1.8	2.0	1.5	0.9	1.0
Bank	10	14.0	1.7	2.5	12.8	1.5	1.4	11.2	1.1	1.0	13.7	1.4	1.7
Age	20	11.6	2.3	2.9	10.9	1.9	1.7	8.8	1.4	1.2	10.3	1.6	1.7
	40	8.0	2.3	2.6	7.3	1.7	1.4	6.5	1.5	1.1	7.5	1.7	1.5
	100	4.3	2.2	2.0	4.3	1.4	1.2	4.2	1.2	0.9	4.9	1.3	1.1
	1000	1.5	1.2	1.1	1.6	0.8	0.7	1.4	0.6	0.5	1.7	0.7	0.8
German	10	19.7	5.6	5.0	21.3	10.3	8.7	19.1	8.2	8.2	20.4	14.2	11.5
age	20	18.1	6.0	4.4	18.6	6.7	6.4	16.7	7.0	7.0	18.8	9.9	9.0
	40	15.9	6.7	4.8	15.0	5.6	4.9	11.7	6.6	5.8	14.9	6.4	6.9
	100	7.9	5.8	3.9	7.5	5.5	3.8	8.2	6.5	4.3	9.1	4.4	4.2
	200	4.2	3.7	3.1	4.4	4.1	3.3	4.7	4.1	3.3	4.7	3.8	3.5
German	10	21.5	10.5	8.2	17.6	7.0	6.3	19.4	8.5	8.6	20.0	5.9	6.5
Gender	20	16.2	10.0	7.8	13.2	7.1	5.1	14.1	8.4	7.8	15.4	5.9	4.9
	40	11.6	9.2	6.6	11.4	8.4	4.5	11.1	7.7	5.9	11.1	6.1	3.8
	100	7.1	6.5	5.4	6.9	6.6	3.7	7.0	6.1	4.8	5.9	6.4	2.8
	200	3.2	3.3	3.0	4.0	4.0	2.9	3.6	3.4	2.9	4.0	4.0	2.2
Compas-R	10	21.1	2.9	4.2	20.7	4.0	4.8	20.3	1.4	2.4	23.1	6.6	8.4
Race	20	14.8	2.8	3.3	15.2	3.9	3.8	15.8	2.0	2.5	16.6	7.8	8.0
	40	11.7	3.0	3.0	12.1	3.9	3.6	11.6	2.0	2.0	10.9	9.9	8.1
	100	6.8	2.9	2.8	7.4	3.7	3.4	8.5	2.1	1.8	7.9	7.7	6.0
	1000	2.0	1.5	1.6	1.9	1.6	1.7	1.9	1.3	1.2	1.9	1.9	1.8
Compas-R	10	21.3	3.8	5.0	22.0	3.4	3.8	23.4	3.5	4.4	25.4	19.1	13.7
Gender	20	18.5	3.8	5.1	18.4	3.3	4.0	17.4	3.3	4.6	21.4	23.8	12.3
	40	12.2	3.4	4.0	13.0	3.0	3.3	13.7	2.8	3.6	15.0	23.8	9.5
	100	8.8	3.2	3.3	9.1	2.7	2.6	8.5	2.1	2.7	9.8	15.5	8.0
	1000	2.0	1.7	1.4	2.2	1.4	1.3	2.4	1.6	1.4	1.9	1.9	1.8
Compas-VR	10	17.4	4.0	4.0	15.6	4.4	4.4	15.7	2.6	2.4	19.7	6.1	6.5
Race	20	13.5	4.7	4.3	13.7	5.0	4.8	13.6	3.3	2.9	15.9	10.7	6.5
	40	9.6	4.5	3.8	9.6	4.5	3.9	9.9	3.1	2.4	11.1	8.8	5.5
	100	5.6	3.6	3.1	5.2	3.8	3.4	6.2	2.6	2.0	6.6	6.8	3.7
	1000	0.9	0.8	0.8	0.9	0.8	0.8	0.9	0.8	0.8	1.1	1.2	0.9
Compas-VR	10	17.2	5.6	5.4	16.8	5.7	5.3	19.0	5.8	6.3	21.3	18.9	9.8
Gender	20	13.3	5.4	5.1	14.1	5.4	4.9	14.0	5.7	6.2	16.0	28.2	8.7
	40	9.3	5.1	4.7	9.7	4.9	4.5	10.5	5.3	5.7	12.4	30.9	6.9
	100	6.4	3.7	3.4	5.9	3.5	3.1	6.3	4.2	4.4	7.1	18.5	4.5
	1000	1.0	0.8	0.9	1.0	0.9	0.9	0.9	0.9	1.0	1.4	0.9	0.9
Ricci	10	17.7	16.1	14.6	14.4	7.5	7.9	12.2	1.9	2.1	13.1	1.7	1.6
Race	20	11.2	11.8	9.8	9.3	7.2	7.1	8.5	1.5	1.5	9.5	2.0	2.1
	30	7.4	7.7	6.5	5.8	5.1	4.6	6.0	1.1	1.1	6.4	1.9	2.0





SENSITIVITY ANALYSIS FOR THE CALIBRATION PRIORS

	Multi-	layer P	erceptron	Logist	ic Regi	ression	Rano	dom Fo	orest	Gauss	Gaussian Naive Bayes		
Method	10	100	1000	10	100	1000	10	100	1000	10	100	1000	$\mu_a \sim N(0, .4\alpha), \sigma_a \sim TN(0, .15\alpha)$
BB	18.52	8.48	2.46	18.74	9.14	2.30	18.24	9.00	2.12	18.88	9.54	2.32	$\dots N(0, 4n) = TN(0, 15n)$
BC, α =0.1	2.63	2.60	2.27	2.46	2.49	2.13	2.87	2.84	2.43	4.67	4.51	0.78	$\mu_b \sim N(0, .4\alpha), \sigma_b \sim IN(0, .15\alpha)$
BC, $\alpha = 0.2$	2.63	2.56	2.08	2.46	2.51	2.06	2.85	2.83	2.09	4.63	3.95	0.82	$\mu_c \sim N(0, 2\alpha), \sigma_c \sim TN(0, .75\alpha)$
BC, $\alpha {=} 0.3$	2.60	2.52	1.88	2.42	2.51	1.95	2.85	2.79	1.86	4.44	3.36	0.97	
BC, $\alpha = 0.4$	2.49	2.46	1.74	2.41	2.57	1.90	2.74	2.82	1.70	4.25	3.06	1.11	
BC, $\alpha {=} 0.5$	2.49	2.38	1.71	2.44	2.60	1.82	2.82	2.77	1.65	4.01	2.86	1.43	
BC, $\alpha {=} 0.6$	2.47	2.37	1.62	2.55	2.62	1.75	2.82	2.88	1.60	3.81	2.79	1.46	
BC, α =0.7	2.61	2.48	1.51	2.36	2.63	1.70	2.90	2.86	1.54	3.54	2.80	1.50	
BC, α =0.8	2.86	2.30	1.47	2.52	2.73	1.63	2.87	2.86	1.46	3.51	2.77	1.60	
BC, α =0.9	2.93	2.27	1.43	2.44	2.82	1.64	2.87	2.90	1.46	3.14	2.91	1.58	
BC, α =1.0	3.05	2.31	1.50	2.71	2.74	1.57	2.99	2.96	1.42	3.31	2.85	1.68	
BC, α =1.1	3.14	2.37	1.45	2.65	2.86	1.55	2.90	3.10	1.40	3.25	3.03	1.65	
BC, α =1.2	3.11	2.19	1.49	2.73	2.80	1.52	3.27	3.01	1.39	3.20	3.03	1.68	
BC, α =1.3	3.48	2.30	1.51	2.91	2.94	1.54	3.11	3.21	1.39	3.15	2.96	1.71	
BC, α =1.4	3.76	2.28	1.47	3.17	3.01	1.51	3.26	3.21	1.30	3.48	3.21	1.75	
BC, α =1.5	3.67	2.20	1.49	3.12	2.94	1.51	3.46	3.05	1.34	3.23	3.19	1.66	
BC, α =1.6	4.06	2.24	1.45	3.26	2.93	1.47	3.56	3.13	1.33	3.48	3.17	1.69	
BC, α =1.7	4.02	2.27	1.46	3.46	3.15	1.46	3.75	3.10	1.27	3.43	3.19	1.74	
BC, α =1.8	4.35	2.14	1.42	3.36	3.09	1.50	3.76	3.26	1.29	3.67	3.22	1.81	
BC, α =1.9	4.35	2.30	1.48	3.48	2.94	1.42	3.54	3.30	1.28	3.82	3.35	1.84	
BC, α =2.0	4.69	2.16	1.44	3.87	2.99	1.54	3.91	3.46	1.21	3.83	3.18	1.81	
BC, $\alpha {=} 5.0$	8.11	2.54	1.63	6.31	3.32	1.53	5.32	4.13	1.31	5.25	3.82	2.13	
BC, α =10.0	10.39	2.63	1.63	7.18	3.83	1.70	7.19	4.41	1.42	6.32	4.08	2.33	



